16.2 Iterated Integrals

Suppose that f is a function of two variables that is integrable on the rectangle *R= [a, b] x [c, d].* We use the notation to mean that x is held fixed and *f(x, y)* is integrated with respect to y from *y=c* to *y=d.* This procedure is called **partial integration with respect to y.** Notice the similarity to the method for partial differentiation.

Now, is a number that depends on the value of *x*, so it defines a function of *x*:

.

If we now integrate *A(x*) with respect to *x* from *x=a* to *x=b*, we get

.

The integral on the right side of the equation is called an **iterated integral.** Typically, brackets are omitted to give

.

This method also works for partial integration with respect to .

**Note:** When integrating iterated integrals, always work the inside out. The inner integral corresponds to the innermost partial derivative. For example, look at the equation below:

As you can see, the innermost integral corresponds to the innermost differential

**Fubini’s Theorem** If f is continuous on the rectangle then

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In the special case where *f(x,y)* can be factored as the product of a function of *x* only and a function of *y* only, the double integral of *f* can be written in a particularly simple form:

where R=[a, b] x [c, d].